

DISPERSION OF AN INSTANTANEOUSLY HEATED MATERIAL AND DETERMINATION OF ITS EQUATION OF STATE FROM THE PRESSURE AND MOMENTUM

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The dispersion of a material heated (for example, by laser radiation) "instantaneously," i.e., in a time much smaller than the characteristic dispersion time, is considered. It is shown that heating of a layer of material even to internal energies much smaller than the heat of evaporation can result in expulsion of part of the material due to the interaction of the rarefaction waves and to the formation of negative stresses. The dependence of the momentum on the applied energy and thermodynamic properties in the event of this "split-off" effect is determined for the cases of uniform and nonuniform heating. It is suggested that instantaneous heating of a material layer be used for determining its equation of state for densities close to the solid-state density.

§1. Several papers concerning the thermodynamic properties of solids and liquids at high pressures and temperatures have appeared recently. A high proportion of studies in this field have been concerned with the equation of state of a material under compression and heating in a powerful shock wave; the relationship among the parameters characterizing the state (i.e., the pressure p , the energy e , and the specific volume v) was established in these studies from the shock adiabats determined by various methods [1, 2]. By the specific nature of shock methods, high pressures and temperatures entail increases in the density of the material. It is also of interest, however, to investigate the thermodynamic functions at high pressures and temperatures, but at normal density ρ_0 (or at nearly normal density). Such states can be achieved by "instantaneous" heating of the material ($\rho = \rho_0$) and its subsequent dispersion ($\rho < \rho_0$).

In fact, if the energy release time τ is smaller than the characteristic time of change of the temperature and density during hydrodynamic motion (from now on we assume that heat conduction is negligible), then the density ρ_0 of the material remains constant during rapid heating and even after it. At this density ρ_0 (or at one close to it) the energy e_x of "cold" compression or expansion and the corresponding pressure p_x are small as compared with the "thermal" energy e_T and the "thermal" pressure p_T . The pressure produced by heating is related to the energy released by the expression

$$p = p_T = \rho_0 e_T (\gamma - 1) = \rho_0 e (\gamma - 1), \quad (1.1)$$

where γ is the effective ("integral") adiabatic exponent. If it is possible to measure the pressure p , then γ can be found from relation (1.1) provided one knows the energy concentration $f(m)$ in a unit mass with the Lagrangian coordinate m .

"Instantaneous" energy release in a material can be produced, for example, by radiation from a Q-switched pulse laser. In this case the energy concentration $f(m)$ can be found from the known absorption κ of the material (or from its known absorption function $\kappa(e_T, \rho_0)$) on the basis of the energy applied per unit surface,

$$f(m) = E \kappa \exp\left(-\int_0^m \kappa dm\right). \quad (1.2)$$

The energy $f(m)$ released per unit mass is easiest to find when $\kappa = \text{const} = \kappa_0$, where κ_0 is the absorption of the material in the "cold" state. Let us find the characteristic mass $m_0 = 1/\kappa_0$ of the heated material.

For sufficiently large values of E/m_0 the energy concentration exceeds the heat of evaporation Q of the material in the surface layer,

$$m \lesssim m_0, \quad f(m) = f(0) = E/m_0.$$

For large values of E/m_0 this is also true for the deeper layers,

$$m_s \gg m_0, \quad Q = f(m_s) \ll E/m_0.$$

By varying the energy E applied per unit area (this can be readily accomplished, for example, by suitably adjusting the laser beam focusing at constant energy), one can investigate a very interesting range of (e_T, ρ) values, i.e., $e_T \approx Q$, $\rho \approx \rho_0$, i.e., the range in which determination of the equation of state by theoretical methods is very difficult. The quantity κ in this range remains constant if absorption occurs on the molecular or atomic level, and if considerable thermal motions of the molecules or atoms, or the excitation of their internal degrees of freedom, do not as yet affect κ .

The simplest picture of heating and subsequent dispersion occurs if the radiation-heated layer is "transparent," i.e., if its thickness $x_r \ll x_0 = m_0/\rho_0$ so that the layer is heated practically uniformly. The condition of "instantaneousness" is fulfilled for characteristic thicknesses $x_r \gg c\tau$, where c is the speed of sound, i.e., if the quantity x_r satisfies the latter restriction for the given value of τ . For values of c close to the speed of sound c_0 in the "cold" state, i.e., for c on the order of 3×10^5 cm/sec and $\tau \approx 2 \times 10^{-8}$ sec (a typical radiation time for a Q-switched pulse laser) this condition is $x_r > 6 \times 10^{-3}$ cm. Thus, the radiation path $x_0 \geq x_r$ must not be too small. This condition is not fulfilled, for example, for metals acted on by radiation in the optical range ($x_0 \approx 10^{-5}$ cm). However, there is a sufficiently broad range of materials which do satisfy this condition. Furthermore, the depth to which the radiation penetrates can often be varied by altering the color of the material.

§2. It is sometimes more feasible to determine the total reactive momentum I instead of measuring p directly. Analysis of the dependence of the momentum I on the incident energy E and study of the dispersion pattern of the material can provide certain information about the state of the latter.

Let us consider a straightforward evaluation of the momentum for the case where the specific energy (i.e., the energy per unit mass) is much larger than the heat of evaporation Q , and where the heated material can be considered as a gas. If we disregard energy redistribution

Among the particles, we can say that all of the energy E/m_0 applied to unit mass, with the exception of the heat Q expended on the disruption of molecular bonds, is converted into the kinetic energy of dispersion $u^2/2$, where u is the velocity of the material (equal for all the particles), and that the mechanical recoil momentum during dispersion of the uniformly heated material of mass m_r is given by the relation

$$I = um_r = m_r \sqrt{2((E/m_0) - Q)} = (m_r/m_0) \sqrt{2Em_0(1 - f_s)}. \quad (2.1)$$

Here and below the dimensionless parameter $f_s = Qm_0/E$. Consideration of energy redistribution over the particles during dispersion due to

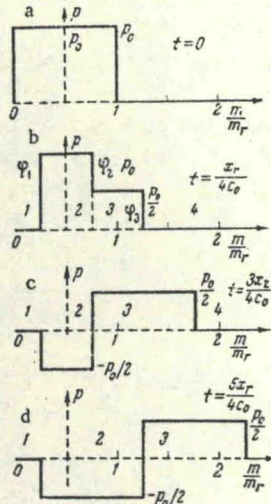


Fig. 1

the work performed by some of the particles on others and allowance for the consequent change in the momentum I gives rise to a coefficient χ in front of the right side of formula (2.1). In the case of adiabatic dispersion of a layer uniformly heated over its mass [3] this coefficient depends on the adiabatic exponent γ , but this dependence is a relatively weak one; thus, as the exponent γ changes from 1 to 3, the coefficient χ changes from 0.798 to 0.865. As we infer from the solution of the problem of dispersion of a gradually ("slowly") heated material [4], the momentum can also be estimated from formula (2.1), but the coefficient χ turns out to be completely independent of the adiabatic exponent γ and is equal to $(2/\pi)^{1/2} \approx 0.798$ in the case of uniform heating.

From this it follows that in investigating the magnitude of the momentum arising with concentrations of released energy which are higher than the heat of evaporation of an instantaneously or gradually heated layer it is difficult to obtain information about the thermodynamic state of the material. As will be shown below, the momentum can arise for energy densities comparable with, and even much smaller than, the heat of evaporation Q . If $f(m) \approx Q$ or $f(m) < Q$, the material cannot be regarded as an ideal gas, and the estimate of the mechanical momentum given below implies that its value is highly sensitive to the adiabatic exponent γ , the speed of sound c , and their dependences on the thermal energy e_T , and that the role of γ is more significant than in the case $f(m) \gg Q$.

We note that for $E = m_0Q$ formula (2.1) yields a momentum of zero. This is because in deriving (2.1) we assumed that expansion of the material following heating presupposed expenditure of energy sufficient to break all the intermolecular bonds determining the structure of the solid. However, the condition $f(m) \leq Q$ does not mean that the material has not evaporated. It simply means that it cannot evaporate completely. Conversely, the inequality $f(m) > Q$ does not mean that the material has evaporated completely, since relief can involve partial condensation, and the heat expended can be smaller than Q .

§3. Let us consider the following gasdynamic picture which can lead to dispersion of the material and

to the formation of a mechanical momentum even in total absence of evaporation, i. e., for $f(m) \ll Q$.

"Instantaneous" heating uniform over the mass m_r produces the pressure p defined by relation (1.1). If the surface-heated material is bounded by a vacuum on one side ($m = 0$) and by cold material on the other ($m = m_r$), then rarefaction waves begin to move (at the initial instant) from the above boundaries into the material. These waves meet inside the heated layer (if the cold and hot layers are of the same material, they meet at the center); moreover, a compression wave begins to move into the interior of the material.

Let us consider the picture of this process. Figure 1a shows the pressure distribution at the initial instant ($p = p_0$ in the $0 \leq m \leq m_r$ range). Figure 1b shows that the heated material has begun to be relieved by the rarefaction waves ϕ_1 and ϕ_2 , and that the shock wave ϕ_3 is propagating in the cold material. If $E \ll m_0Q$, then the amplitude Δp of the resulting waves is smaller than $p_0c_0^2$, since the ratio Q/c_0^2 is on the order of unity. This allows us to use the acoustic approximation. Then, if we assume that the speed of sound remains constant throughout the heating process, we can write

$$\Delta p = \rho_0 c_0 \Delta u. \quad (3.1)$$

Let us introduce the characteristic velocity $u_0 = p_0/\rho_0 c_0$. Applying formula (3.1) to the waves ϕ_1 , ϕ_2 , and ϕ_3 , we find that in domain 1 the material is relieved ($p = 0$) and moves leftward with the velocity $u = -u_0$. In domains 2 and 4 it is stationary, while in domain 3, where $p = p_0/2$, its velocity is $u = u_0/2$. The waves ϕ_1 and ϕ_2 move towards each other with the velocity c_0 . They meet at the center of the hot layer at the instant $t_0 = x_0/2c_0$. The material in domain 2 (Fig. 1c) now moves with the velocity $u = -u_0/2$, and is at the pressure $p = -p_0/2$. If the resulting stress exceeds the dynamic tensile strength σ of the material, then rupture occurs, and the entire domain 1, i. e., the layer of mass $m_r/2$, splits off with the velocity $u = -u_0$, producing the momentum

$$I = \frac{u_0 m_r}{2} = \frac{m_r p_0}{2 \rho_0 c_0} = \frac{p_0 x_r}{2 c_0}. \quad (3.2)$$

Taking a different approach (i. e., assuming that the two halves of the layer are acted on by the pressure p_0 throughout the time t_0), we obtain the same expression for I . Making use of the relationship between the initial pressure p_0 and the initial volume energy concentration E_0/x_0 defined by Eq. (1.1), we find that the momentum I depends on the energy E_r released in the layer x_r by the expression

$$I = \frac{E(\gamma - 1)}{2c_0} \frac{x_r}{x_0} = \frac{E_r(\gamma - 1)}{2c_0}. \quad (3.3)$$

If the material is not uniformly heated, i. e., if the total mass of the layer is $M \approx m_0$ or $M \gg m_0$, then, by virtue of the rapid (exponential for $\kappa = \text{const}$) decrease of energy with depth, we can schematize the phenomenon and assume that in the material in the layer $m_r = m_0$ is heated, and that this layer contains all of the energy E , while the layer $m > m_0$ is absolutely cold.

Then $E_T = E$, and Eq. (3.3) implies that the momentum does not depend on the depth of penetration x_0 of the radiation.

If there is no split-off, the picture becomes that shown in Fig. 1d. Here the shock wave ($p = p_0/2$, $u = u_0/2$) is followed by the rarefaction wave ($p = -p_0/2$, $u = -u_0/2$) which ends in a jump after which $p = 0$, $u = 0$. Since the width of the "positive" phase domain ($u > 0$) and the width of the "negative" phase are equal to each other and to the initial thickness x_r of the heated layer, it follows that the total momentum of the system is zero.

Recalling that Q is of the same order of magnitude as c_0^2 and introducing λ according to the relation $c_0^2 = \lambda Q$, we can rewrite Eq. (3.2) in a form which differs from Eq. (2.1) only by a coefficient dependent on f_s ,

$$I = \frac{m_r}{m_0} \sqrt{2Em_0} \frac{(\gamma-1)}{2\sqrt{2\lambda f_s}}. \quad (3.4)$$

From Eq. (3.4) we infer that for $f_s \geq 1$ the momentum is not equal to zero as Eq. (2.1) would indicate. The momentum decreases slowly with increasing f_s . As we see from Eq. (3.3) and Eq. (3.4), the momentum depends rather strongly on γ .

§4. Formulas (3.2) and (3.3) can be refined by taking account of the changes in the speed of sound in the material as it is heated. The speed of sound in some materials for $\rho = \rho_0$ and for heats on the order of Q has been measured directly [1, 2, 5, 6]. These data were obtained by shock wave compression of a material whose density has been reduced in advance (i. e., a material in porous or powder form). The quantity γ was also determined in these experiments. Since it is difficult to produce high-velocity shock waves in such experiments, the latter are few in number and have been performed for a small range of materials only. Let us therefore estimate the changes in the value of c with heating.

In general form, the equation describing both the condensed and the gaseous states of a material, as well as its "intermediate" states, can be written as

$$p = p_x + p_r, \quad e = e_x + e_r, \quad p_r = (\gamma-1)\rho e_r$$

$$e_x(\rho) = - \int_{v_0}^v p_x dv = \int_{\rho_0}^{\rho} (p_x / \rho^2) d\rho. \quad (4.1)$$

Here $\gamma(e_T, \rho)$ is the effective adiabatic exponent; p_x , p_r and e_x , e_r are the "elastic" and "thermal" components of the pressure and energy.

Let us make the simplifying assumption that $\gamma = \gamma(s)$, i. e., that γ does not vary or varies little along the isentrope. As we see from [5, 6], the existing experimental findings do not contradict this assumption. The equation of state of the material can then be written as

$$p = p_x + (\rho / \rho_0)^\gamma F(s) = p_x + (\rho / \rho_0)^\gamma p_r^\circ. \quad (4.2)$$

Here F and p_r° (the thermal pressure for $\rho = \rho_0$) are functions of the entropy s .

Differentiating Eq. (4.2) and making use of relation (1.1), we obtain the following expression for the speed of sound:

$$c^2 = \left(\frac{dp}{d\rho} \right)_s = \frac{dp_x}{d\rho} + \left(\frac{dp_r}{d\rho} \right)_s = c_0^2 + \gamma \frac{p_r}{\rho} =$$

$$= c_0^2 + \gamma(\gamma-1)e_r = c_0^2 \left(1 + \frac{\gamma(\gamma-1)e_r}{\lambda Q} \right). \quad (4.3)$$

According to this expression the speed of sound increases with heating. For $e_T = Q$, if $\gamma = 2$ and $\lambda = 1$, the speed of sound increases by about $\sqrt{3}$ times. Taking

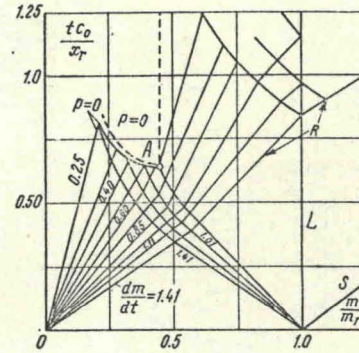


Fig. 2

account of relation (4.3), we can rewrite formula (3.2) for the momentum as

$$I = \frac{\gamma-1}{2} \frac{E}{c} \frac{x_r}{x_0} = \frac{(\gamma-1)E_r}{2c_0 \sqrt{1 + \gamma(\gamma-1)/\lambda f_s}}. \quad (4.4)$$

For large heats, when $E \gg m_0 Q$, i. e., when $f_s \ll 1$, this formula can be written as

$$I = \frac{m_r}{2m_0} \frac{\sqrt{\gamma-1}}{\sqrt{2\gamma}} \sqrt{2Em_0} = \frac{m_r}{m_0} \chi_0 \sqrt{2Em_0}. \quad (4.5)$$

Expression (4.4) differs from Eq. (2.1) (for $f_s \ll 1$) only by a numerical coefficient χ_0 of order 1/2. The difference can be explained as follows: in deriving formula (4.4) we did not allow for the fact that upon encountering the rarefaction waves the pressure drops to zero or to negative values which do not result in immediate rupture of the layer, i. e., the rarefaction waves (which are not acoustic) are of finite width, and that the layer heated to the "gaseous" state is dispersed completely, instead of one-half of it splitting off as in the case of low heats. We note that if the heated material is bounded by cold, but more rigid, material, i. e., if the displacement of the boundary is negligible, then the entire heated layer "splits off," and the coefficient in Eq. (4.4) and Eq. (4.5) must be doubled. Numerical computations carried out by the method of characteristics for the problem of dispersion of a uniformly heated layer bounded by a vacuum on one side and by cold material of the same density on the other showed that with heating to $f(m) \approx Q$ the additional momentum following encounter of the rarefaction waves at the center is negligible prior to the instant, while the point at which the pressure is $p = 0$ or $p = -\sigma$ is close to $m_r/2$. Figure 2 is a general diagram showing interaction of the centered rarefaction waves, formation of the shock wave, and "split-off" in the specific case $e_T = Q$, $\lambda = 2$, $\gamma = 2$. Here A is the point where the pressure $p = 0$ is first achieved; R is the boundary of the rarefaction wave; S is the shock wave; $0 < m/m_r < 1$ is the hot layer; $m/m_r > 1$ is the cold layer; $m = 0$ is the vacuum; L is the layer boundary. In carrying out such computations one must, of course, choose a specific form of the equation of state.

We assumed that $p_x(\rho)$ is given by the expression

$$p_x(\rho) = \rho_0 c_0^2 \frac{y^n - y^k}{n - k} \quad \left(y = \frac{\rho}{\rho_0} \right). \quad (4.6)$$

We therefore assumed that the repulsive and attractive forces are described by power functions. The elastic energy can then be written as

$$e_x(\rho) = c_0^2 \left(\frac{y^n - 1}{n-1} - \frac{y^k - 1}{k-1} \right). \quad (4.7)$$

From Eq. (4.7) we see that the binding energy Q of the solid (e_x for $y \rightarrow 0$) is related to c_0^2 by the expression

$$\lambda = \frac{c_0^2}{Q} = \frac{n-k}{(n-1)(k-1)}. \quad (4.8)$$

Using the Eq. (4.8), we can determine the relationship between n and k from the known value of λ . The values of n and k can also be determined separately,

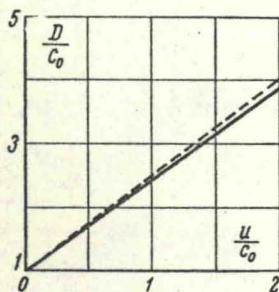


Fig. 3

for example, from the known shock adiabat $p_N(\rho)$, by virtue of the condition of its tangency to the cold compression curve $p_X(\rho)$ and the coincidence of their second derivatives (for $\rho = \rho_0$). We note that it is not always possible to satisfy this condition and condition (4.8).

Experimental data can usually be represented as the linear dependence of the velocity D of the shock wave front on the velocity u of the material behind the shock wave [1, 2],

$$D = c_0 + \beta u. \quad (4.9)$$

If we assume that relation (4.9) is valid for any u/c_0 , we can express the adiabat in the form

$$p_N = \frac{\rho_0 c_0^2 (y-1)y}{[\beta - (\beta-1)y]^2}. \quad (4.10)$$

From Eq. (4.6) and Eq. (4.10) we infer that for $y = 1$

$$\frac{dp_x}{dy} = \frac{dp_N}{dy} = \rho_0 c_0^2, \quad \frac{d^2 p_N}{dy^2} = 2(2\beta - 1) \rho_0 c_0^2, \quad \frac{d^2 p_x}{dy^2} = (n+k-1) \rho_0 c_0^2. \quad (4.11)$$

Thus, $n+k = 4\beta - 1$. Setting $n = 3$ and $k = 2$, we find that $\lambda = 2$ and $\beta = 3/2$, which are typical average values for a broad range of materials. For $n = 3$ and $k = 2$ we have

$$p_x = \rho_0 c_0^2 (y^3 - y^2), \quad e_x = \frac{1}{2} c_0^2 (y-1)^2. \quad (4.12)$$

We note that near $y = 1$ the elastic energy e_x can always be written in the form (4.12), since it has a maximum at the equilibrium point. In [7] it is shown that the coefficients of the subsequent terms of the expansion in $(y-1)$ are small. Equation (4.12) is sufficiently typical, and the qualitative conclusions which can be drawn from it are valid for other functions $p_X(\rho)$ as well.

From Eq. (4.12) and Eq. (4.10) for $\rho = \rho_S$, i.e., for $y = y_S$, we obtain

$$\gamma(\rho_S) = 2 \frac{4-y_S}{5-y_S} + 1. \quad (4.13)$$

Here ρ_S is the density of the material at the shock wave front (at the instant when it passes through the given particle) which is related single-valuedly to the entropy s .

This expression was already used by Koryavov [7], who assumed, however, that $\gamma = \gamma(\rho)$, i.e., that the function $\gamma(\rho)$ is of the same form as the function $\gamma(\rho_S)$. If this were true, then for $\rho = \rho_0$ we would have the constant value $\gamma = \gamma_0$ regardless of the amount of heating (according to Eq. (4.13) we have $\gamma_0 = 2.5$).

It is clear, however, that γ decreases both with heating of a material at constant density and with heating along the shock adiabat. Both $\gamma = \gamma(s)$ and $\gamma = \gamma(\rho_S)$ do, in fact, conform to this condition. It is clear that strong heating (when $\rho_S \approx \rho_*$) corresponds to the limiting value of γ_* on the shock adiabat. According to Eq. (4.10), for $\beta = 3/2$ the limiting compression is $\rho_* = 3\rho_0$, while according to Eq. (4.13) we have $\gamma_* = 2$. We note that for $u \geq c_0$ the linearity of $D(u)$, i.e., the linearity of Eq. (4.9), can, in fact, be violated, and the limiting value of γ_* can differ from the value of γ determined on the basis of β . There is not enough information available on the thermodynamic states of various materials at $\rho = \rho_0$ to enable us to determine the true function $\gamma(\rho_0, s)$ and to establish to what extent this function differs from (4.13). In analyzing the equation of state we shall make one further simplifying assumption, i.e., that $\gamma = \text{const}$. For $\gamma = 2$ and $p_X(\rho)$ as given by Eq. (4.12), the shock adiabat assumes a form which differs slightly from Eq. (4.10) for $\beta = 3/2$,

$$p_N(\rho) = \rho_0 c_0^2 \frac{(y^2 - 1)y}{3 - y}. \quad (4.14)$$

The function $D(u)$ corresponding to Eq. (4.14) (the solid curve in Fig. 3) is, as before, close to straight line (4.9) with $\beta = 3/2$ (the dashed curve in Fig. 3).

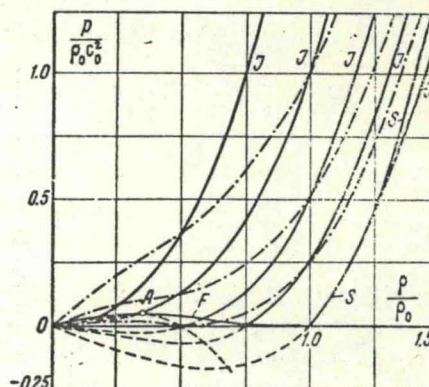


Fig. 4

Thus, the equation of state ($p_X(\rho)$) as defined by (4.12) used in actual numerical computations of hydrodynamic motion, and the value of γ given by Eq. (4.13), or $\gamma = \text{const} = 2$, yield shock adiabats of fairly typical shape, so that the conclusions drawn from these computations must be sufficiently general.

§5. The solid curves in Fig. 4 represent the shock adiabat S (4.14) and the isentropes I (4.2) for

$\gamma = 2$; the dot-dash curves represent the constant thermal energy e_T . Extension of the isentropes below the boundary of the two-phase domain F are indicated by the broken curves (A is the critical point). The "cold" curve $p_X(\rho)$ has a minimum point. The corresponding values of $p_{X \min}$ and $\rho_{\min}$ can be found (with $p_X(\rho)$ defined by Eq. (4.6)) from the condition

$$\rho_{\min} = \rho_0 \left(\frac{k}{n} \right)^{1/(n-k)}, \quad p_{X \min} = -\frac{\rho_0 c_0^2}{n} \left(\frac{k}{n} \right)^{k/(n-k)} \quad (5.1)$$

which for $n = 3$ and $k = 2$ yields the expressions

$$\rho_{\min} = \frac{2}{3}\rho_0, \quad p_{\min} = -\frac{4}{27}\rho_0 c_0^2. \quad (5.2)$$

The above value of $p_{\min}$ yields the theoretical tensile strength σ_T of the material. As we see, σ_T is approximately one order smaller than $\rho_0 c_0^2$. The real strength σ of the material is, of course, considerably smaller than σ_T .

The above equation of state has a curve of metastable states ($dp/d\rho = 0$ for $e_T = \text{const}$) is represented by a broken curve in Fig. 4). This curve intersects the straight line $p = 0$ for the first time at $\rho = \rho_M$ (for $\rho > \rho_M$ the curve of metastable states lies in the domain $p < 0$),

$$\rho_M = \rho_0 \left(\frac{k-1}{n-k} \right)^{\frac{1}{n-k}}. \quad (5.3)$$

For $n = 3$ and $k = 2$ the quantity $\rho_M = \rho_0/2$. As we see from formula (4.2) and Fig. 4, the isentrope which passes through this point emerges from the point $\rho = \rho_0$ and $e_T = Q = c_0^2/2$ (for $n = 3$ and $k = 2$ we have $\lambda = 2$). For $p = 0$ and $\rho = \rho_M$ the "cold" energy $e_X = Q/4$, and the thermal energy $e_T = Q/2$, while, as noted above, the initial thermal energy (for $\rho = \rho_0$ on the same isentrope) is $e_T^0 = Q$.

Thus, if the material cannot withstand negative stresses, then the energy expended on adiabatic relief of a given particle prior to rupture of the material ($p = 0$) is much smaller than Q (for $\rho > \rho_M$ or $e_T^0 \leq Q$).

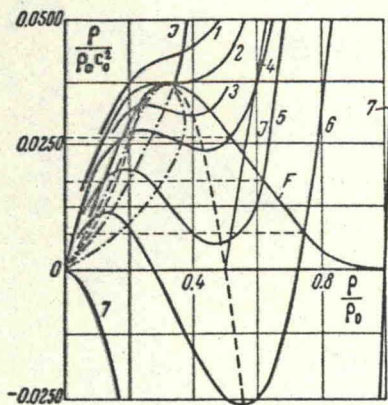


Fig. 5

Figure 4 shows the boundary of the two-phase domain. In the above determination of the point of intersection of the isentrope with the straight line $p = 0$ and in our numerical computations of the hydrodynamic motion we assumed that the isentrope can be extended all

the way to the curve of metastable states or $p = 0$ without taking cognizance of the fact that the end point lies in the domain of a possible two-phase state in which the material can experience stratification into two phases (in equilibrium), i.e., that the liquid is superheated in some domain. However, this is unimportant from the standpoint of energy expenditures (this is clear from Fig. 4), so that the major portion of the work of expansion and overcoming the cohesive forces is expended below the line delimiting the two-phase domain. This is valid only for isentropes which enter the two-phase domain far from the critical point on the boiling curve. This is because fulfillment of the conditions of thermodynamic equilibrium of the vapor and liquid means that the slope of the isentrope changes in this domain.

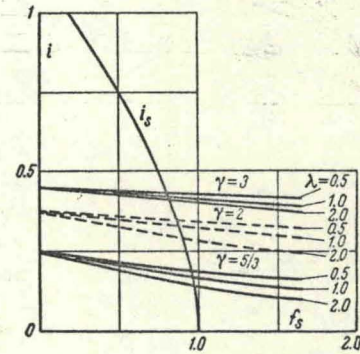


Fig. 6

The boundary $p = p_g(v)$ of the two-phase domain can be found from the ordinary condition. It is the line connecting the points which satisfy the condition

$$\int_{v_1}^{v_2} p(v, e_T = \text{const}) dv = p_g(v_2 - v_1). \quad (5.4)$$

Here p_g is some pressure, and v_1 and v_2 are points on the boundary of the two-phase domain. The critical point is given by the ordinary relation

$$\frac{dp}{d\rho} = 0, \quad \frac{d^2p}{d\rho^2} = 0 \quad \text{for } e_T = \text{const}.$$

This implies that

$$\rho_k = \rho_0 \left(\frac{k(k-1)}{n(n-1)} \right)^{\frac{1}{n-k}}, \quad p_k = \rho_0 c_0^2 \left(\frac{k(k-1)}{n(n-1)} \right)^{\frac{k}{n-k}} \left(\frac{k-1}{n} \right) \quad (5.5)$$

for $\gamma = \text{const}$.

Let $n = 3$ and $k = 2$. Then $\rho_k = \rho_0/3$ and $p_k = \rho_0 c_0^2/27$.

The domain of values p, ρ close to the critical point is shown in greater detail in Fig. 5. Here I is the isentrope and F is the boundary of the two-phase state domain; the dot-dash curves represent states with an equal liquid content in the equilibrium vapor-liquid mixture; the dashed curve represents the metastable states. We denote the ratio e_T/c_0^2 by e_T^* . Points 1-7 correspond to the values of $e_T^* = 0.35, 1/3, 0.32, 0.30, 0.26, 0.20$, and 0 .

For $p = p_k$ and $\rho = \rho_k$ the quantity e_X is equal to $4Q/9$. In other words, nearly one-half of the binding energy is already expended on overcoming the cohesive forces even before the isentrope enters the two-phase

domain (the thermal energy at the critical point is still high, $e_T = 2Q/9$). We note that the isentrope passing through the critical point also passes through the point $\rho = \rho_0$, $e_T = 2Q$, and $p = \rho_0 c_0^2$. If the material is heated and expands nonadiabatically, but rather by continuous heating, then it is necessary to maintain $e_T > 2Q/3$, so that the curve $p(\rho)$ passes over p_K outside the two-phase domain during expansion.

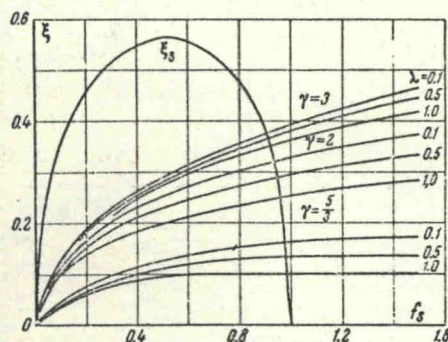


Fig. 7

Hence, in order to ensure that the material is essentially vaporized and rendered gaseous during expansion, it is necessary to provide an energy release in it of between Q and $2Q$.

Now let us compare instantaneous heating at constant density with heating of a solid material in a shock wave. The initial point of the relief adiabat which passes through the critical point and lies on the Hugoniot adiabat (4.14) has the parameters $p_S = 10.7\rho_0 c_0^2$, $\rho_S = 2.2\rho_0$, while that which passes through the point $p = 0$, $\rho = \rho_M$ has the parameters $p_S = 6\rho_0 c_0^2$, $\rho_S = 2\rho_0$. The above estimates and the estimates of [1] imply that substantial evaporation requires that the amplitude of the shock wave be quite large (e.g., for $\rho_0 c_0^2 = 3 \times 10^5$ kg/cm² it is necessary to have a pressure $p = 2-3 \times 10^6$ kg/cm²). Such pressures are attainable in the laboratory [1, 2], although with some difficulty. It is therefore possible in principle to compare data on the value of γ obtained for instantaneous heating with data obtained for heating of the solid material in a shock wave followed by relief.

§6. Let us introduce the dimensionless momentum by means of the relation

$$i = I / \sqrt{2Em_0}. \quad (6.1)$$

Figure 6 shows i as a function of the parameter $f_s = Qm_0/E$ characterizing the degree of heating of the material for the case of a uniformly heated layer of mass $m_T = m_0$. The momentum I was computed from formula (2.1) for the gaseous layer i_s and from formula (3.3) (for various values of the parameters γ and λ). The coefficient χ in formula (2.1) which corrects for energy redistribution during dispersion was taken to be 0.8 (see §2). We also introduce the ratio of the momentum to the energy and render it dimensionless by multiplying it by $\sqrt{Q}$.

$$\xi = I \sqrt{Q} / E = i \sqrt{f_s}. \quad (6.2)$$

The dependence of the "energy utilization" factor ξ introduced this way is shown in Fig. 7 for the case of a uniformly heated layer of thickness $m_T = m_0$. The factor ξ obtained in computing I from formula (2.1) is denoted by the subscript s ; that determined from Eq. (3.3) is denoted (in the text) by the subscript 0. As we see, the factor ξ_s has a maximum, while the quantity ξ_0 is close to the maximum value of ξ_s .

As we noted in §3, the interaction of acoustic rarefaction waves in the center of a uniformly heated layer produces a maximum negative stress equal to $p_0/2$ in absolute value. Introducing the dimensionless criterion w (the ratio of the internal energy e_w , heating to which results in rupture of the material as a result of rarefaction wave interaction in the heated layer, to Q), we can write the following expression for the case of uniform heating:

$$w = e_w / Q = 2\sigma(\gamma - 1) / Q\rho_0. \quad (6.3)$$

If we assume that the strength σ of a material is equal to its theoretical value σ_T , then for $n = 3$ and $k = 2$ we have $\sigma_T = \rho_0 c_0^2 / 7 = 2\rho_0 Q / 7$, and for $\gamma = 2$ the parameter $w_T = 0.56$. In fact, the real strength of a material is considerably smaller than its theoretical strength (one must, of course, allow for increases in real strength as a result of decreases in loading time [8, 9]). Here we need only note that $w \ll w_T \approx 0.5$, i.e., that $w \ll 1$, and that a mechanical momentum arises as a result of rupture of the material even with heating to energies e_w considerably smaller than Q .

The assumptions which lead to (2.1) and (3.3) are not valid in the range $e \approx Q$ (in the former case the material cannot be considered gaseous; in the latter case one cannot neglect the momentum arising after encounter of the rarefaction waves and the possibility of numerous split-offs and dispersion of the entire material). The function ξ in this domain can be obtained by means of the aforementioned numerical

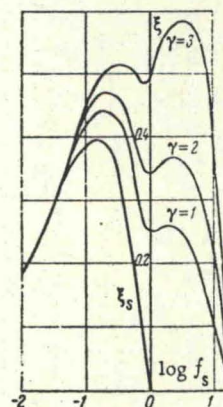


Fig. 8

computations of the entire gasdynamic picture of motion. The possible divergence of the results of such computations (for large heats) from the approximate estimate (3.3) depends essentially not only on parameters such as γ , λ , w , but also on the equation of state of the material in the domain $\rho < \rho_0$ (relief follows encounter of the rarefaction waves). For an equation of

state having the form assumed in §§4 and 5, the momentum (according to numerical computations of the hydrodynamic motion) is fairly close to (3.3) (the disparity for $f_s \leq 1$ is not larger than 20–30%, and diminishes with decreasing heating). In our computations we assumed that stratification into two phases did not occur, and that rupture of the material took place with relief to $p = 0$. To determine ξ and I more precisely in the range $f(m) \approx Q$ it is necessary to compute the motion of the material using a more exact equation of state and to allow for the possible formation of two phases; in addition, the moving material can take the form of a mixture of vapor and droplets (or solid fragments) not in thermal equilibrium with each other, i.e., having different velocities and temperatures.

From now on we shall estimate the momentum from (3.3) in the range $f_s \geq 1$ and from (2.1) in the range $f_s \leq 1$.

From Figs. 6 and 7 we see that the value of the "split-off" momentum I_0 is essentially determined by the thermodynamic properties of the heated and cold material, while the boundary where such a momentum still exists depends on the strength of the material relative to "split-off."

§7. When laser radiation interacts with a sufficiently thick layer of material, the radiation flux diminishes with thickness as a result of absorption (see §2). This makes it necessary to alter formulas (2.1) and (3.3) to allow for nonuniform energy release with respect to depth. Assuming that the final dispersion velocity of each gas particle is independent of those of the other particles, we can rewrite formula (2.1) as

$$I_s = \chi \int_0^{m_x} \sqrt{2(f(m) - Q)} dm. \quad (7.1)$$

Here I_s is the momentum of the gaseous layer; $f(m)$ is the energy released per unit mass of the gas (which varies with depth); m_x is the mass of the layer of dispersed "gaseous" material, i.e., the mass up to the point at which we cease to employ formula (7.1) (as we shall show below, m_x turns out to be close to the mass m_s of the layer in which the energy released exceeds the binding energy Q). Formula (7.1) was proposed by E. M. Rabinovich, who set $m_x = m_s$ and $\chi = 0.8$. The coefficient χ in the case of nonuniform heating corrects for energy redistribution not only during dispersion of the material into the void, but also during propagation of the compression and shock waves occasioned by the reaction force exerted by the dispersed material and propagating into the interior of the cold material. Redistribution by the shock wave can be important only in the case of a strong wave ($\Delta p \gg \rho_0 c_0^2$) which travels far and takes in a large mass (the "brief" shock effect [1]).

From now on we shall assume a relatively low degree of heating and shall neglect the redistribution of energy by the shock wave. Redistribution of energy during dispersion of the gas into the vacuum in the case of nonuniform heating of the mass results in a value of χ different from 0.8. But this difference is small [4] in the case of an exponential function, for example; thus, in (1.2) for $\chi = \text{const} = 1/m_0$ in the case where

$m_x \gg m_0$ the coefficient $\chi \approx 0.6$ (with prolonged heating, when redistribution of energy by the shock wave is slight [4], and when energy is redistributed only during dispersion into the vacuum). Since we are interested primarily in the case where $m_x \approx m_0$ or $m_x \ll m_0$ (and finally, where $m_x = 0$), the gaseous layer is uniformly heated, and χ can be taken equal to 0.8 as before.

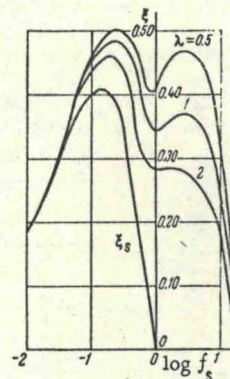


Fig. 9

We note that according to [4] the momentum arising with dispersion of the gaseous layer varies little with increasing ratio of the time τ of energy application to the characteristic layer dispersion time, i.e., our estimate of the momentum is valid even for a very thin layer of the above description.

We can transform formula (3.3) in similar fashion:

$$I_0 = \frac{\gamma - 1}{2} \int_{m_x}^{m_w} \frac{f(m)}{\sqrt{c_0^2 + \gamma(\gamma - 1)f(m)}} dm. \quad (7.2)$$

Here I_0 is the momentum of the "split-off" layer, and m_w is the mass of the layer to the point of "split-off," i.e., the point at which heating is insufficient for rupture of the material to occur. We note that if we neglect the variation of the speed of sound c relative to c_0 , we have

$$I_0 = \frac{(\gamma - 1)(E_w - E_x)}{2c_0}, \quad (7.3)$$

where E_x and E_w are the energies present in the entire layer from $m = 0$ to $m = m_x$ and to $m = m_w$, respectively; E_w is the energy of the layer from m_x to m_w . In computing the momentum I_0 in the case of nonuniform heating, we determined the parameter w and the mass m_w of the "split-off" layer from condition (6.3) and the condition $f(m_w) = wQ$, which we justified above for the case of a uniformly heated layer. According to the solution of the problem of thermoelastic deformations (in the hydrodynamic approximation), the pressure distribution with exponential heating is

$$\begin{aligned} p &= \frac{1}{2} p_0 (e^{\mu} + e^{-\mu}) \text{ for } \mu > 0 \\ p &= \frac{1}{2} p_0 (e^{\mu} - e^{-\mu}) \text{ for } \mu < 0 \\ \mu &= (m - \rho_0 c_0 t) / m_0, \quad \eta = (m + \rho_0 c_0 t) / m_0. \end{aligned} \quad (7.4)$$

There is an abrupt (stepwise) decrease in pressure at the curve $\mu = 0$. The negative stress of maximum absolute value occurs at this moving point and is given by

$$p_{\min} = -\frac{1}{2} p_0 (1 - \exp(-2m / m_0)). \quad (7.5)$$

It diminishes with time from 0 to $-p_0/2$. It is clear that solution (7.4), (7.5) is valid until, according to Eq. (7.5), $p_{\min} > -\sigma$. For $p_0 \approx 2\sigma$ the occurrence or nonoccurrence of split-off, the magnitude of the expelled mass m_w , and the value of the momentum I_0 depend on random factors. On the other hand, if $p_0 \gg 2\sigma$,

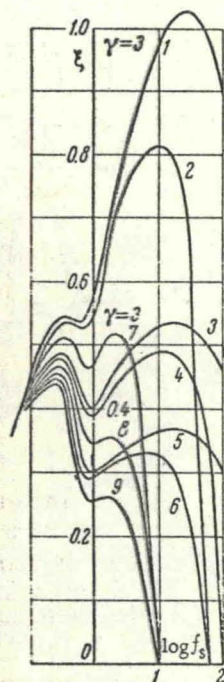


Fig. 10

then not only single, but multiple split-off and fragmentation of the material will occur. With an exponential decrease of the degree of heating with depth, the mass m_w which is split-off is approximated by the relation

$$m_w = m_0 \ln(E/wm_0Q) = m_0 \ln(1/wf_s). \quad (7.6)$$

The exact value of the mass m_w has little effect on the value of the momentum, since according to formula (7.3), the quantity which determines the momentum is the energy absorbed between the "transition" point m_x (close to the "sublimation" point m_s) and the split-off point m_w . In fact, $\sigma \ll \rho_0 Q$, so that $f(m_w) \ll f(m_s) = Q$, so that the major portion of the energy beyond the evaporation point lies between the points m_x and m_w . Because of the rapid decrease in $f(m)$ with depth, changes in the position of m_w have little effect on the magnitude of this energy, and therefore on the value of the momentum. We determined the point of transition from formula (7.1) to (7.2) on the basis of the assumption of continuous momentum of an infinitely thin unit layer, i.e., from the assumption that the integrands of (7.1) and (7.2) are equal,

$$\chi \sqrt{2(f(m_x) - Q)} = \frac{(\gamma - 1)f(m_x)}{2\sqrt{e_0^2 + \gamma(\gamma - 1)f(m_x)}}. \quad (7.7)$$

Computations using Eq. (7.7) (see also Figs. 6 and 7) indicate that the energy per unit mass $f(m_x)$ at the "transition" point m_x is close to the binding energy Q . The total momentum I can be determined by adding the momentum I_s of the layer of material heated to the

"gaseous" state (given by Eq. (6.3)) to the momentum I_0 of the layer of material partially vaporized or fragmented (given by Eq. (7.1)) under the assumption that these quantities are independent ($I = I_s + I_0$). In reality, however, the processes of dispersion of the two phases can interact. For example, the presence of a positive pressure at the boundary with the gaseous phase can hinder the formation of negative stresses in the condensed medium (this fact was drawn to our attention by G. F. Filippov). But if the speed of sound in the gaseous layer is markedly higher, and if its thickness is markedly smaller than the speed of sound in, and the thickness of, the "cold" layer, then the time of dispersion of the gaseous layer and the time required for the pressure in it to drop to negligibly small values are considerably smaller than the characteristic time of gasdynamic processes and of "split-off" in the "cold" layer. The momentum of the gaseous layer is simply transmitted through the region where the material is heated to energies smaller than Q .

Conversely, if the maximum amplitude of the waves passing through the "cold" layer is sufficiently large as compared with $\rho_0 c_0^2$, then the latter is "pumped" with additional energy, which results in a further increase in momentum. However, we are concerned with relatively low degrees of heating where $\Delta p \ll \rho_0 c_0^2$, and the "gaseous layer" affects the cold layer negligibly, if at all.

The results of our computations of the momenta for the case of an exponential function $f(m)$ for constant values of the parameters characterizing the equation of state of the material (γ and λ) and its strength (w) appear in Figs. 8, 9, and 10 in the form of the "energy utilization" factor with allowance for "split-off" ($\xi = \xi_0 + \xi_s$) and without allowance for it (ξ_s) as functions of the parameter f_s characterizing the degree of heating of the material. As we see from Fig. 8, the changes in momentum with changes in γ are quite marked, so that the momentum of a nonuniformly heated layer can be used as a basis for drawing conclusions about the thermodynamic properties of the material. From Fig. 10 we see that with nonuniform heating the dynamic tensile strength (the change in w) affects not only the boundary at which the "split-off" momentum arises, but also the magnitude of this momentum.

Curves 1, ..., 9 in Fig. 10 correspond to the following values of the parameters (γ, w): 1 (3, 0.002), 2 (3, 0.015), 3 (2, 0.002), 4 (2, 0.01), 5 (1.67, 0.002), 6 (1.67, 0.015), 7 (3, 0.1), 8 (2, 0.1), 9 (1.67, 0.1). If the quantity γ varies with heating (and therefore with depth), then in comparing experimental and theoretical values we can determine only the average value of γ for the heated layer. It is therefore desirable to conduct experiments under conditions of uniform heating.

§8. The above notions concerning the production of a mechanical momentum at low specific energies and the relationship of its value to the equation of state, the strength of the material, and the applied energy required experimental verification, which was, in fact, obtained for several materials. We investigated the mechanical momentum, the dispersion pattern, the velocity of the split-off material, and the pressure arising directly in the material under a laser beam. The results of these experiments will be published later. We merely note at this time that the production of a "split-off" momentum predicted by one of the authors of the present paper and the quantitative estimate of this momentum obtained above were confirmed.

It should be noted that the phenomenon under discussion is of interest not only as a source of data on the thermodynamic properties of a body. The effects of instantaneous heating by laser radiation and the subsequent dispersion of the material are of independent interest. They can be considered, for example, as a method of producing surface explosions in a whole range of materials for the purpose of generating

shock and compression waves of very small length—something which is difficult to achieve by other methods.

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THE DOUBLE LAYER IN AN INSULATOR SUBJECT TO SHOCK COMPRESSION

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It has been shown [1] that a polar dielectric produces charges on the surfaces perpendicular to the direction of propagation of a shock wave; a current passes in the circuit of Fig. 1, in which 1 is the fixed plate with guard ring, 2 is the moving plate, and 3 is the insulator, the direction of incidence being shown by the arrows.

The initial current step is proportional to the dipole moment of the molecule, the number of dipoles per unit volume, and the plate area S , while it is inversely proportional to the thickness l_0 . The current is produced by rotation of the dipoles.

The effect in, say, lucite can be used in dynamic pressure transducers. Here we assume that a double layer is formed behind the shock-wave front; the equivalent circuit is discussed.

1. EQUIVALENT CIRCUIT OF A POLARIZATION TRANSDUCER

We assume [1] that the wave polarizes the material, e.g., by rotating the dipoles, but that the state persists only while a dipole is accelerated, with subsequent relaxation and return to an equilibrium state, which has altered thermodynamic parameters. A polarized layer thus exists around the front.

A double layer moving at a constant speed in the circuit of Fig. 1 is equivalent to an emf E_0 determined by the thickness δ of the layer, the degree of orientation of the dipoles, and so on. This source, in general, possesses an internal resistance R_1 , since in the steady state it cannot produce a current greater than a value governed by the number of dipoles oriented in unit time.

Figure 2 shows the $x-t$ diagram, in which OA is the path of the plate (mass speed u), OB is the path of the shock wave (double layer, wave speed D), region 1 is uncompressed material (dielectric constant ϵ_1 , bulk specific resistance ρ_1), and region 2 is compressed material (ϵ_2 and ρ_2).

We consider the case of $\delta \ll l_0$; then for any time $0 \leq t \leq l_0/D$ the system of Fig. 1 may be interpreted via the circuit of Fig. 3a. Switch T is closed at $t = 0$ and opens at $t = l_0/D$. The circuit parameters with subscripts 1 and 2 relate respectively to regions 1 and 2 in Fig. 2 and are written for unit area in the form

$$C_1 = \frac{\epsilon_1}{4\pi(l_0 - Dt)}, \quad C_2 = \frac{\epsilon_2 \sigma}{4\pi Dt}, \quad R_1 = \rho_1(l_0 - Dt), \\ R_2 = \frac{\rho_2 Dt}{\sigma} \quad \left(\sigma = \frac{D}{D-u} \right) \quad (1.1)$$

in which σ is compression. The circuit of Fig. 3a is described by a second-order linear differential equation, which is difficult to solve. We consider two particular cases, in which R_1 is such that its shunt action on C_1 can be neglected.

Case 1. The conductivity behind the shock front is small. We neglect the shunting action of R_2 and pass from the equivalent circuit of Fig. 3a to that of Fig. 3b, in which

$$R = R_1 + R_e, \quad C = C_1 C_2 (C_1 + C_2)^{-1}. \quad (1.2)$$

For this we have

$$E_0 = iR + V_c, \quad \text{or} \quad I + \varphi = 1 \\ (\varphi = V_c E_0^{-1}, I = iR E_0^{-1}). \quad (1.3)$$

We differentiate the equation relating charge to voltage for a capacitor to get

$$I = \varphi R \left(\frac{C}{\varphi} \frac{d\varphi}{dt} + \frac{dC}{dt} \right). \quad (1.4)$$

Here the reciprocal of the quantity in parentheses can be considered as the effective resistance of the capacitor, which is denoted by R_c ,

while R_c/R is denoted by $2\mu^2$. From (1.4), $R_c = dt/dC$ if $\varphi = \text{constant}$. From Eqs. (1.1) and (1.2),

$$\mu = 2 \sqrt{\pi} \frac{l_0 - (1-z)Dt}{\sqrt{2RD\epsilon_1(1-z)}} \quad \left(z = \frac{\epsilon_1}{\epsilon_2 \sigma} \right). \quad (1.5)$$

Substitution of Eq. (1.4) into Eq. (1.3) and replacement of t by μ from Eq. (1.5) gives

$$\frac{d\varphi}{d\mu} - \left(2\mu + \frac{1}{\mu} \right) \varphi = -2\mu. \quad (1.6)$$

The general solution to this is

$$\varphi = \mu \exp(\mu^2) [\text{const} - \sqrt{\pi} \text{erf}(\mu)]. \quad (1.7)$$

The value of μ for $t = 0$ is

$$\mu_0 = 2 \sqrt{\pi} l_0 [2RD\epsilon_1(1-z)]^{-1/2} \quad (1.8)$$

and a current step arises in the circuit, but the potential difference across the capacitor at this instant is $V_c \sim \varphi = 0$. This gives us the constant of integration in Eq. (1.7), so the particular solution of Eq. (1.6) is

$$\varphi = \mu \exp(\mu^2) \sqrt{\pi} [\text{erf}(\mu_0) - \text{erf}(\mu)]. \quad (1.9)$$

It is clear from Eqs. (1.5) and (1.8) that μ_0 and μ are related by

$$\mu = \mu_0 [1 - (1-z)X] \quad (X = Dt l_0^{-1}) \quad (1.10)$$

in which X is a dimensionless coordinate ranging from 0 to 1 as t varies, μ running from μ_0 to $\mu_0 z$.

Finally, from Eq. (1.3) we have

$$I = 1 - \mu \exp(\mu^2) \sqrt{\pi} [\text{erf}(\mu_0) - \text{erf}(\mu)]. \quad (1.11)$$

Figure 4 shows $I = f(X)$ as solid lines 1-5 for μ_0 of 0.5, 1, 2, 4, and 10 respectively.

Case 2. The material behind the front is a good conductor, i.e., $R_{c2} \gg R_2$ and $R = R_1 + R_e \gg R_2$.

Here again the processes in Fig. 2 are described by the circuit of Fig. 3b, and the equations remain the same, except that the capacitance C is the C_1 of Fig. 3a. This alters the form of

$$\mu_0 = 2 l_0 \sqrt{\pi} (2RD\epsilon_1)^{-1/2} \quad (1.12)$$

and Eq. (1.10) becomes

$$\mu = \mu_0 (1 - X) \quad (X = Dt l_0^{-1}). \quad (1.13)$$

The range in μ is from μ_0 to 0 as X goes from 0 to 1; Fig. 4 (broken lines) shows the results from Eq. (1.11) for this case.

In both cases the value $Z = 0.5$ has been used.

2. ANALYSIS OF RESULTS

1) The shape of the $I = f(X)$ curves in Fig. 2 is readily explained, since at $t = 0$ the following operations are performed simultaneously in the circuit of Fig. 3b: the source of emf is switched on and the effective resistance of C drops from $R_c = \infty$ to some finite value R_{c0} . The emf produces a current step $i = E_0/R$ ($I = 1$), with subsequent decay governed by the relaxation time. Further, the change in R_c produces a current step $i = E_0 (R + R_c)^{-1}$ or in dimensionless terms,

$$I = (1 + 2\mu^2)^{-1} \\ (I = (2\mu^2)^{-1} \text{ for } 2\mu^2 \gg 1). \quad (2.1)$$

The shape of the $I(X)$ curves is determined by the joint action of these two factors.

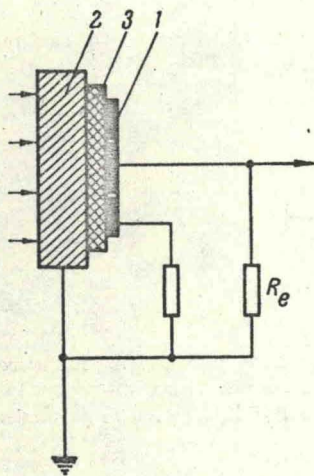


Fig. 1

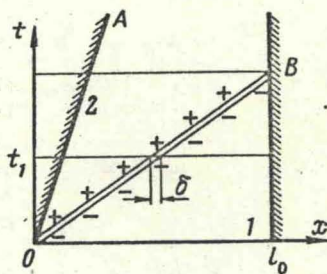


Fig. 2

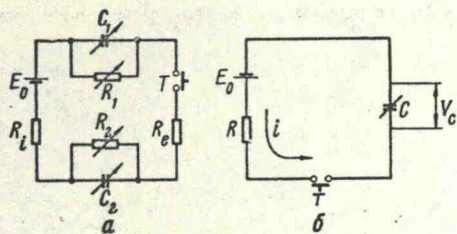


Fig. 3

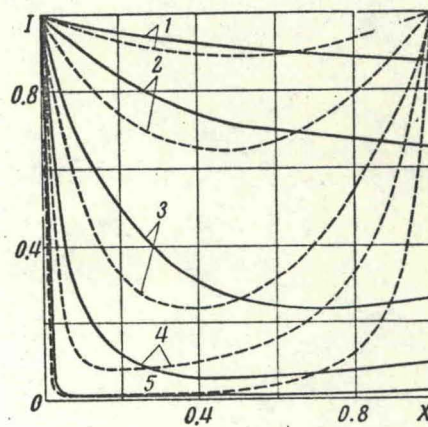


Fig. 4

If RC_0 is small relative to l_0/D (μ_0 large), the first factor may be neglected to a first approximation. Then the initial condition for Eq. (1.7) will be $\varphi = 1$ at $t = 0$, so

$$I = 1 - \mu \exp(\mu^2) (\mu_0^{-1} \exp(-\mu_0^2) + \sqrt{\pi} [\operatorname{erf}(\mu_0) - \operatorname{erf}(\mu)]) \quad (2.2)$$

We stop the series expansion of $\operatorname{erf}(\mu)$ at terms of the second order in μ to get

$$I = (2\mu^2)^{-1}$$

which agrees with (2.1).

2) We put the ratio of RC_2 and R_2 from Eq. (1.1) as $4\pi t/\rho_2 \epsilon_2$.

Case 1 corresponds to $4\pi t/\rho_2 \epsilon_2 \ll 1$, while case 2 corresponds to $4\pi t/(\rho_2 \epsilon_2) \gg 1$. We assume for case 1 that $t = l_0/D \approx 1 \mu\text{sec}$, while $\epsilon_2/4\pi$ is 0.5; then $\rho_2 \gg 2 \times 10^6 \text{ ohm} \cdot \text{cm}$. The condition $4\pi t/\rho_2 \epsilon_2 \gg 1$ for the second case can be obeyed only for $t > t_0$, in which t_0 is some minimum time, which we may reasonably take as the resolving time of the apparatus (about 10^{-8} sec), and so $\rho_2 \ll 2 \times 10^4 \text{ ohm} \cdot \text{cm}$ is not covered by solutions 1 and 2.

3) Consider the region of μ_0 large for case 1. The circuit current is given by Eq. (2.1). By specifying the ratio of the currents for two given values of X (or t) on each of the $I(X)$ curves of Fig. 4, we get an expression for z :

$$z = \frac{1}{4K-1} (1 \pm 2\sqrt{K}) \quad (2.3)$$

in which K is the ratio of the current at $X = 1$ to that at $X = 0.5$; $K > 1$ for this range μ_0 (Fig. 4), while z is positive, so Eq. (2.3) may be put as

$$z = (2\sqrt{K} - 1)^{-1}. \quad (2.4)$$

4) The inverse relation of the initial current step to l_0 [1] becomes an inverse-square relation for μ_0 large as (2.1) shows.

5) The second formula of (2.1), put in the form $i = E_0 dC/dt$, is the basic formula for the capacity-transducer method [2].

This feature may be of value in determining ϵ_2 for dielectrics that do not produce an adequate E_0 , e.g., nonpolar dielectrics or polar ones subject to only small σ (for instance, the lower limit of detection for lucite is given [1] as a pressure of 40 kbar).

Naturally, an external source of emf is needed in these cases.

6) The solution for the circuit of Fig. 3a can, in principle, give information on ρ_2 , the specific bulk resistance behind the shock wave.

Then ϵ_2 is readily found from Eq. (1.5). In case 2 the parameters to be determined are E_0 and R . Figure 4 shows that K is $(I_{X=0.5})^{-1}$, which Eq. (1.11) shows to be uniquely related to μ_0 , and the latter defines R via Eq. (1.12). There is no difficulty in finding E_0 .

7) This solution for substances with orientation polarization is correct also for substances with ionic or atomic polarization, since no assumption is made about the nature of the effect responsible for the emf.

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